Quantum Event Learning and Gentle Random Measurements

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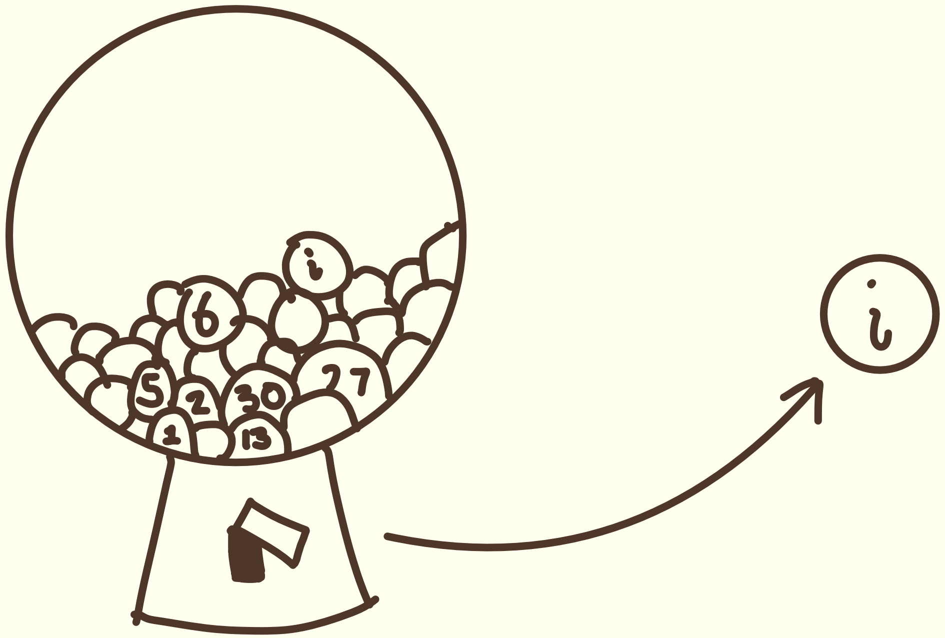
with Adam Bene Watts

Background

and notation

Imaginary scenario

Say you have a machine that spits out a measurement randomly from a set of measurements.



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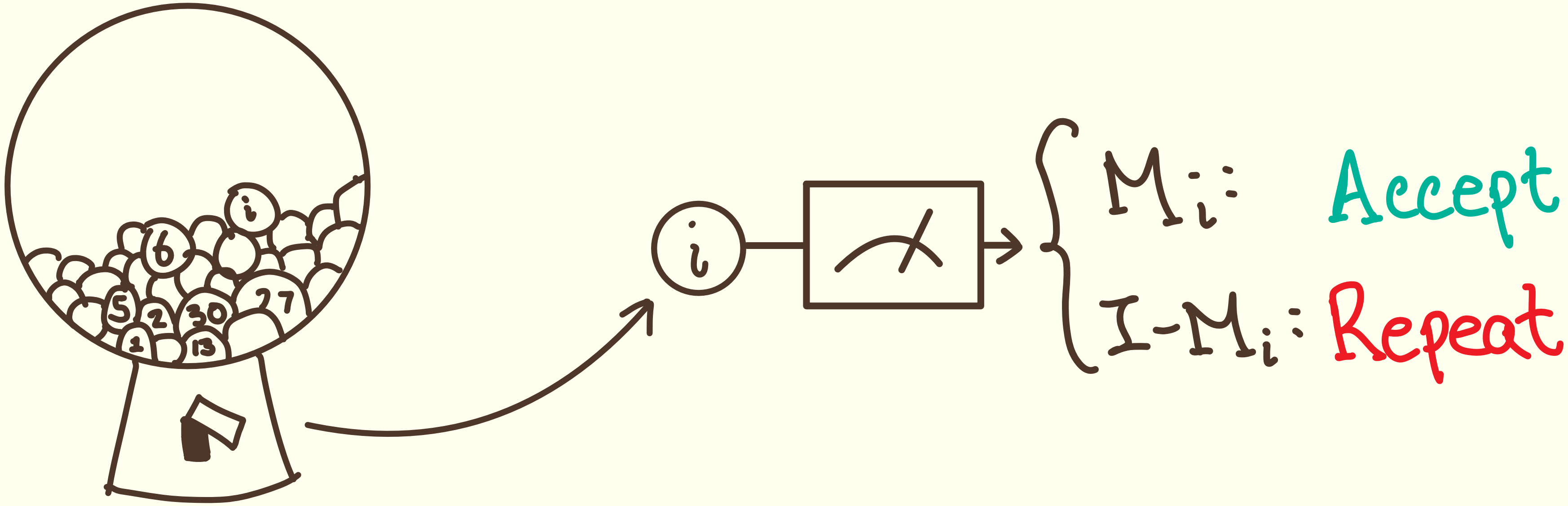

When you see a measurement M_i, you measure $\{M_i, \mathrm{id} - M_i\}$, and “accept” if you see outcome M_i.

Imaginary scenario (Random Measurements)

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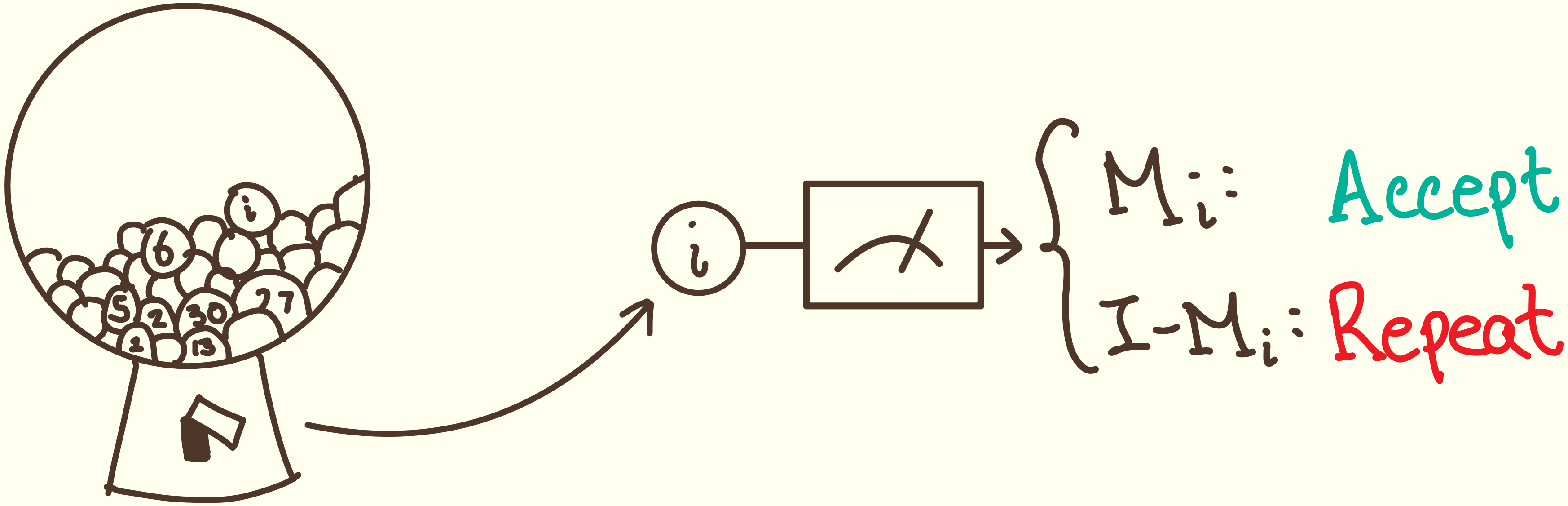

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A useful scenario

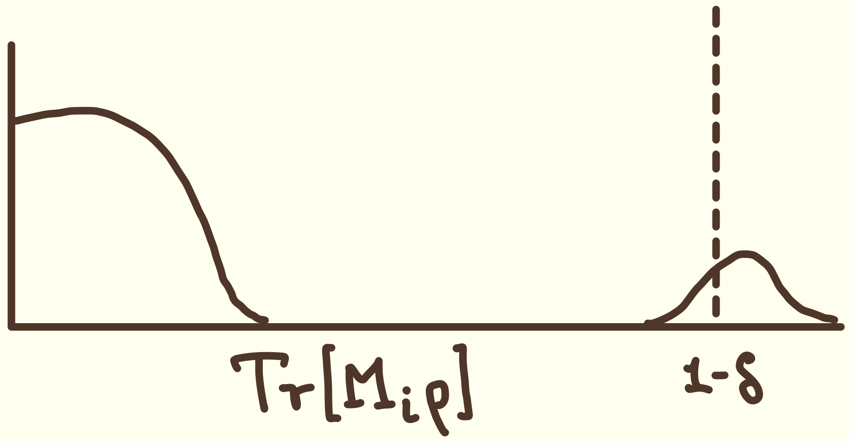
This scenario appears in many places!

Quantum OR

Want to know if, given many measurements, if one will accept, or if all of them will reject on some state.

(one copy)

$$\{M_i\}_{i\in[m]},\ \rho \mapsto$$



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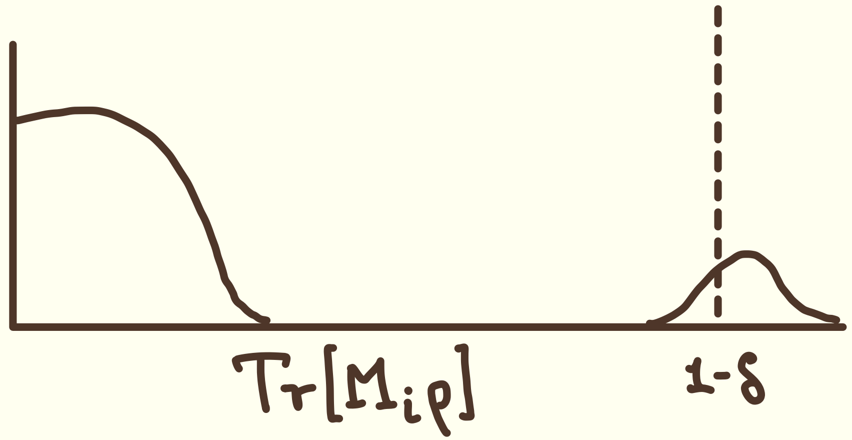

or

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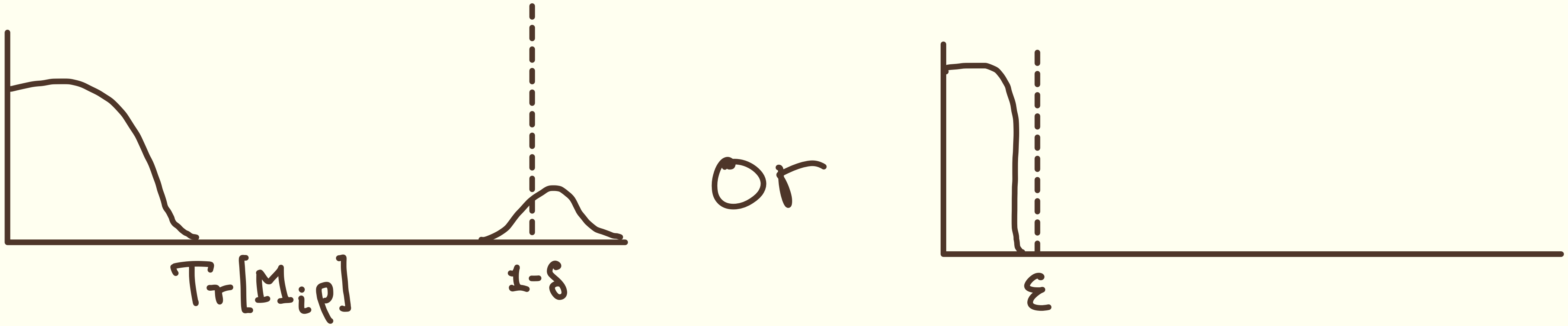

Will random measurements yield the correct answer?

m many

Back to the imaginary scenario

We want to understand properties of the scenario, namely:

If you never accept, how far will your state go from where it started, in expectation, in trace distance?

Back to the imaginary scenario

Two effects seem to be pulling us towards different answers:

1. The gentle measurement lemma
2. The anti-Zeno effect

The gentle measurement lemma

Gives a bound on the disturbance caused by a measurement as a function of the accepting probability of that measurement.

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The gentle measurement lemma

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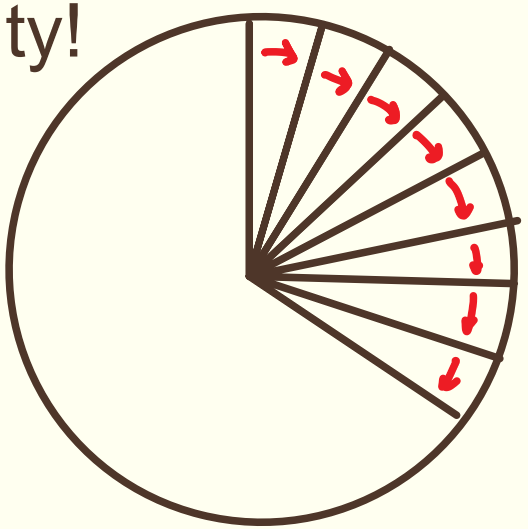
$$\left\| \rho - \rho|_{(I-M_i)(I-M_j)} \right\|_{Tr} \not\leq 2\sqrt{Tr[(M_i+M_j-M_iM_j)\rho]}$$

NOT PSD!

Doesn't apply here because the product of PSD matrices is no longer PSD!

The anti-Zeno effect

Applying a carefully chosen sequence of measurements can cause un-bounded damage while having an arbitrarily small accept probability!



$$\rho_0 = |0\rangle$$

$$M_j = \left(\cos(\varepsilon j)|0\rangle + \sin(\varepsilon j)|1\rangle\right)$$

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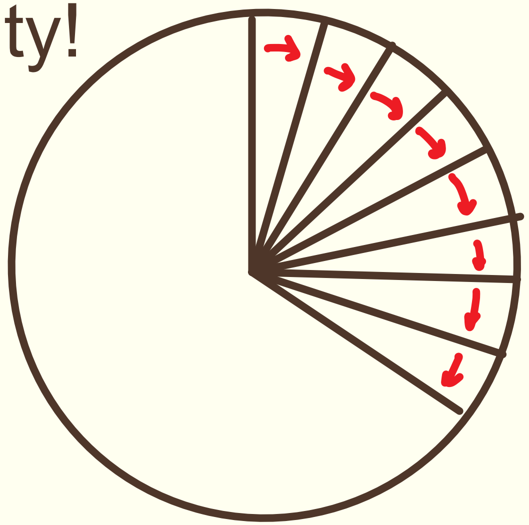

$$\rho_0 = |0\rangle$$

$$M_j = (\cos(\varepsilon j)|0\rangle + \sin(\varepsilon j)|1\rangle)$$

But these sequences seem to require being carefully sequenced, a random sequence shouldn't be anti-Zeno with high probability!

Results

and applications

Gentle random measurements

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State after k many rejects in the random measurement process.

Probability that the random measurement process accepts in the first k measurements.

Randomized quantum OR

Applying random measurements to a single copy of an unknown state is a better quantum OR algorithm than previously known!

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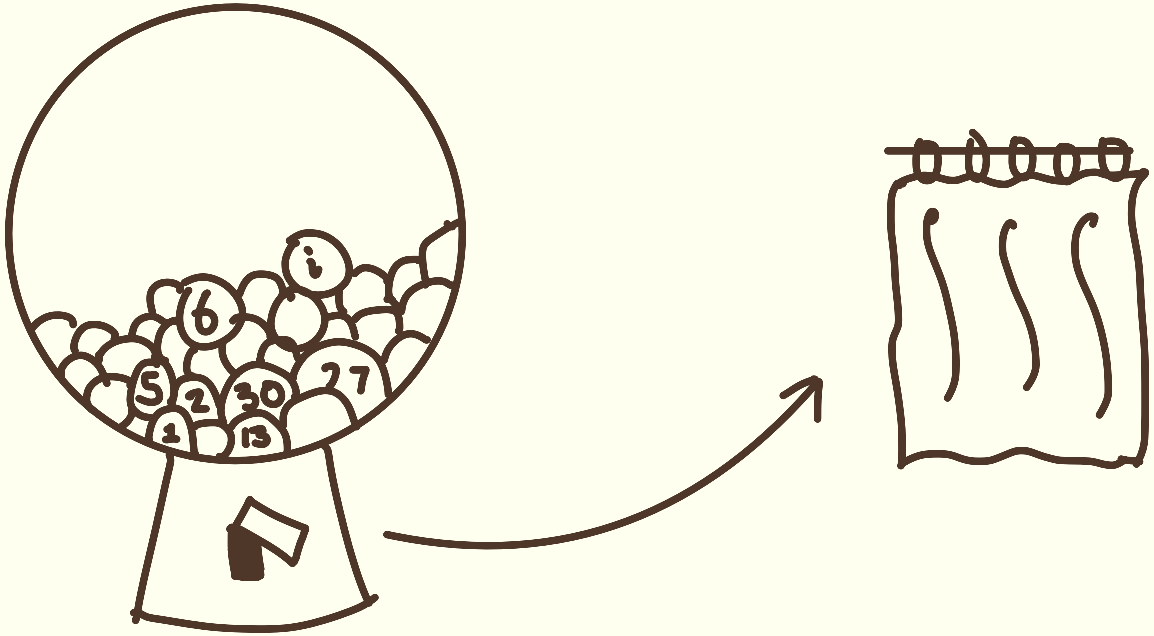
2). If not, gentle random measurements tells us we were likely to have already accepted!

Proof techniques

and blended measurements

Proving the random measurement lemma

Our main technical contribution is to identify a “coherent analog” of random measurements called blended measurements.



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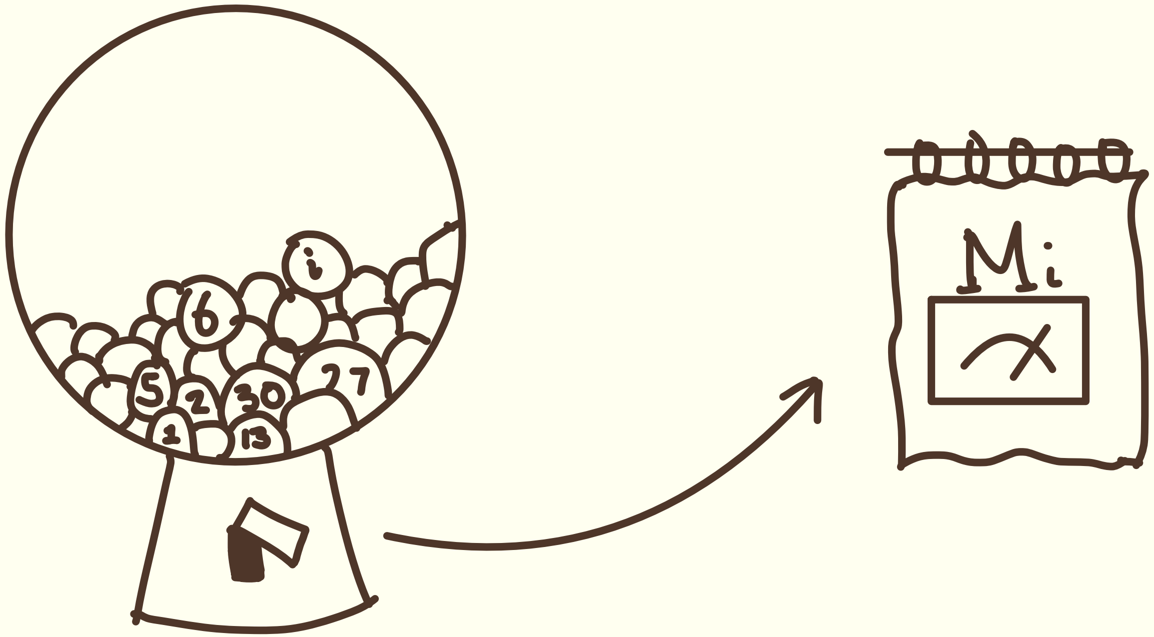

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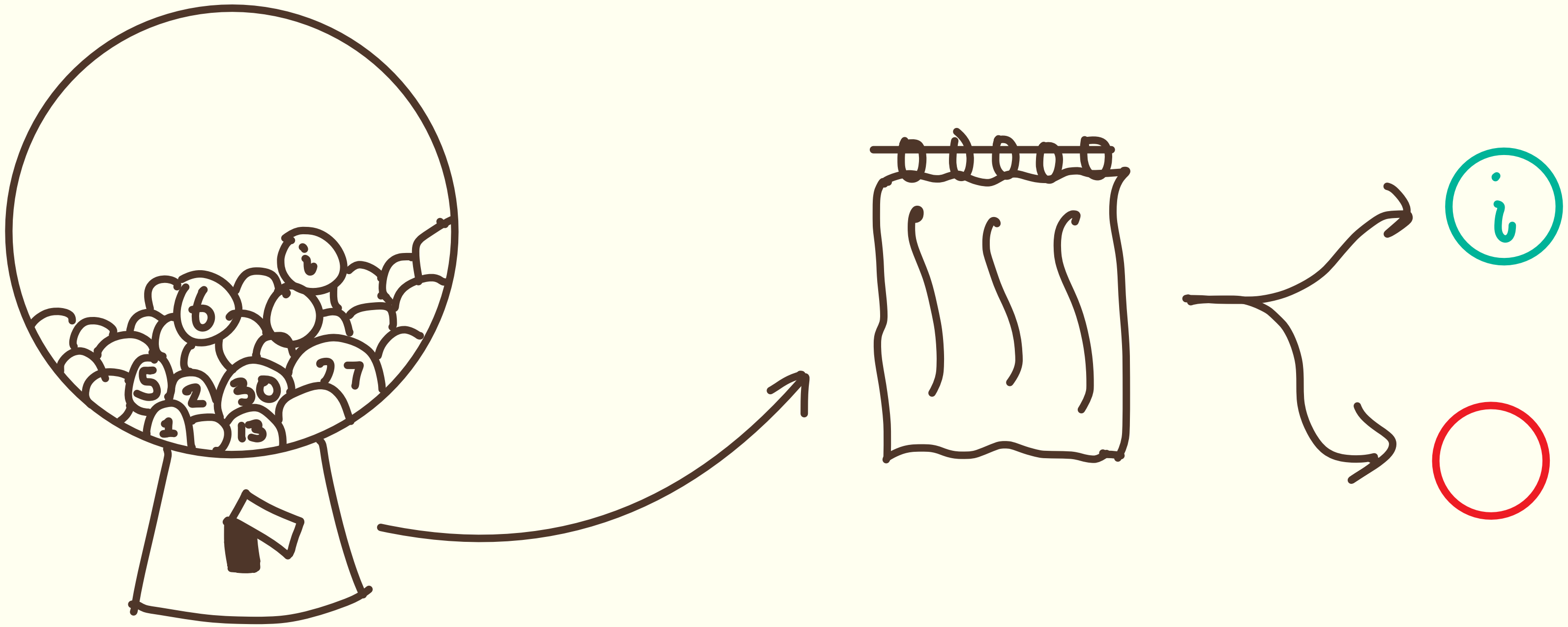

Blended measurements

Let $\{M_i\}_{i\in[m]}$ be a collection of measurements, the blended measurement is:

$$E_0 = \sqrt{I - \sum_i M_i/m}$$

$$E_i = \sqrt{M_i/m}$$

Blended measurements

Repeated blended measurements obey the gentle measurement lemma, almost by definition.

E_0^2 PSD, $(I-M_i)(I-M_j)$ not!

Random measurements

Random measurements can be related to blended measurements as follows:

$$F(\rho, \rho^{(k)}) > 1 - \mathrm{Accept}_B(k)$$

k random measurements

k blended measurements

Random measurements

Random measurements can be related to blended measurements as follows:

$$\mathrm{Accept}(k) \geq \frac{1}{2}\,\mathrm{Accept}_B(2k)$$

k random measurements

2k blended measurements

Blended measurements

Blended measurements on their own are a useful construction in quantum information theory, and if you use them, you can get even better bounds than random measurements!

Open questions

1. Can you prove a gentle random measurement lemma for more general measurements? We only prove it for projective measurements.
2. Can you find other applications of random measurements?
3. Improve on threshold search algorithms, maybe by adding the Laplace mechanism back into the procedure?

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Thanks for Listening!

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